QcBits: constant-time small-key code-based cryptography

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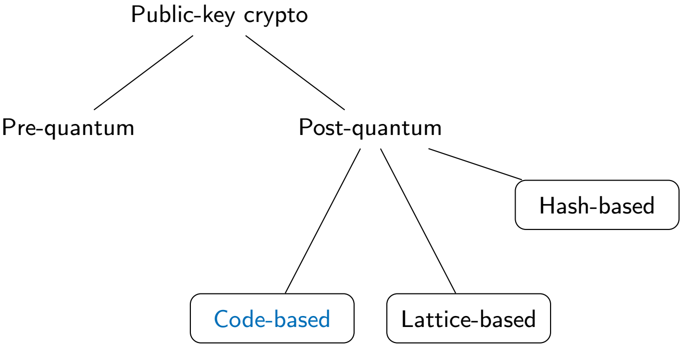
Code-based crypto



Coding theory

Code

- a linear subspace in $\mathbb{F}_2^N$
- can be defined by a parity-check matrix H, e.g.,

$$C = \{c \mid Hc = 0\}$$

Decoding

- compute e (or c) given $c + e$, where e is of weight $\leq t$
- compute e given the syndrome $He = H(c + e)$

Code-based encryption

- McEliece versus Niederreiter

	plaintext	ciphertext
McEliece	c	$c + e$
Niederreiter	e	$H^* e$

- General shape

McEliece/Niederreiter + **some code**

Binary Goppa versus QC-MDPC codes

1976	Public-key crypto
1977	RSA
1978	McEliece + Binary Goppa Codes
1996	NTRU encrypt
2013	QC-MDPC McEliece

Binary Goppa versus QC-MDPC codes

1976 Public-key crypto

1977 RSA

1978 McEliece + Binary Goppa Codes

(≈ 100-KB Niederrieter keys)

1996 NTRU encrypt

2013 QC-MDPC McEliece

(≈ 1-KB Niederrieter keys)

QcBits

2013 QC-MDPC McEliece

QcBits

2013	•	QC-MDPC McEliece
	•	on FPGAs (CHES)
2014	•	on FPGAs, microcontrollers (PQCrypto, DATE)
2015	•	on Haswell CPUs (ACM-TECS)
2016	•	on microcontrollers (PQCrypto)

QcBits

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- **QcBits (new)**

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	QcBits (new)

The problem is timing attacks.

- PQCrypto 2014: constant-time operations assuming no caches
- QcBits: constant-time for a wide-variety of 32/64-bit platforms

Performance results

platform	key-pair	encrypt	decrypt	reference	scheme
Haswell	784 192	82 732	1 560 072	**(new) QcBits**	KEM/DEM
	14 234 347	34 123	3 104 624	ACMTECS 2015	McEliece
Cortex-M4	140 372 822	2 244 489	14 679 937	**(new) QcBits**	KEM/DEM
	63 185 108	2 623 432	18 416 012	PQCrypto 2016	KEM/DEM
	148 576 008	7 018 493	42 129 589	PQCrypto 2014	McEliece

Cycle counts for key-pair generation, encryption, and decryption for 80-bit pre-quantum security. Numbers in RED are non-constant-time. Numbers in BLUE are constant-time.

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$$H = \begin{pmatrix} H^{(0)} & H^{(1)} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \end{pmatrix} \in \mathbb{F}_2^{n \times 2n}$$

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Statistical decoding (bit flipping)

Start with $v = c + e$.

$$Hv = \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \end{pmatrix} v = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

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+) \\
\hline
u = \begin{pmatrix} 2 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 2 \end{pmatrix} \in \mathbb{Z}^{2n}
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$$

Flip v_i if u_i is "large". Repeat until $Hv = 0$.

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Flip v_i if u_i is "large". Repeat until $Hv = 0$.

- parity$=0$: perhaps no errors. no information.
- parity$=1$: one score for each possible position.

Syndrome computation: polynomial view

$$f, g \in \mathbb{F}_2[x]/(x^n - 1)$$

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$$s = v^{(0)} f + v^{(1)} g \in \mathbb{F}_2[x]/(x^n - 1)$$

Sparse-times-dense polynomial in $\mathbb{F}_2[x]/(x^n - 1)$

Compute $vf \in \mathbb{F}_2[x]/(x^n - 1)$

- v dense: array of 32/64-bit words
- f sparse: $i_1, i_2, \ldots,$ where $f_i = 1$

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QcBits computes vf as

$$x^{i_1}v \ + \ x^{i_2}v \ + \ \cdots$$

- Each $x^i v$ is simply a rotation of v.
- Summation can be carried out by XOR instructions.
- Constant-time rotations?

Constant-time rotations (barrel shifter)

Task: compute $v \ggg s$, where $s = (s_1 s_2 s_3 \ldots s_\ell)_2$.

v

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$$\begin{tikzcd}
v \arrow[r] \arrow[d, "s_1 = 0"'] & v \ggg 2^{\ell-1} \arrow[dl, "s_1 = 1"] \\
v' &
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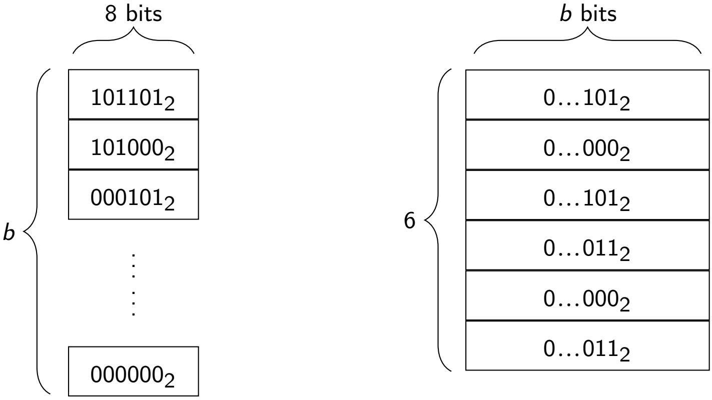
$$u = (sf,\ sg) \in \mathbb{Z}[x]/(x^n - 1)$$

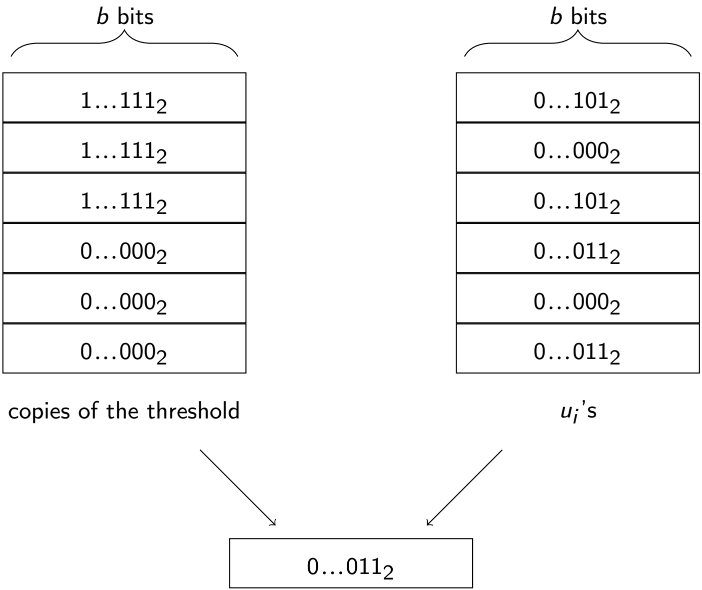
Accumulating $x^i s$



Non-bitsliced

Bitsliced

Flipping bits



Failure rate

Problem

- Can the adversary exploit decoding failures?
- QcBits: $< 10^{-8}$ for 2^{80} security; worse for higher security levels

Solution?

- Julia Chaulet, Nicolas Sendrier. "Worst case QC-MDPC decoder for Mceliece cryptosystem". ISIT 2016.

www.win.tue.nl/~tchou/qcbits/